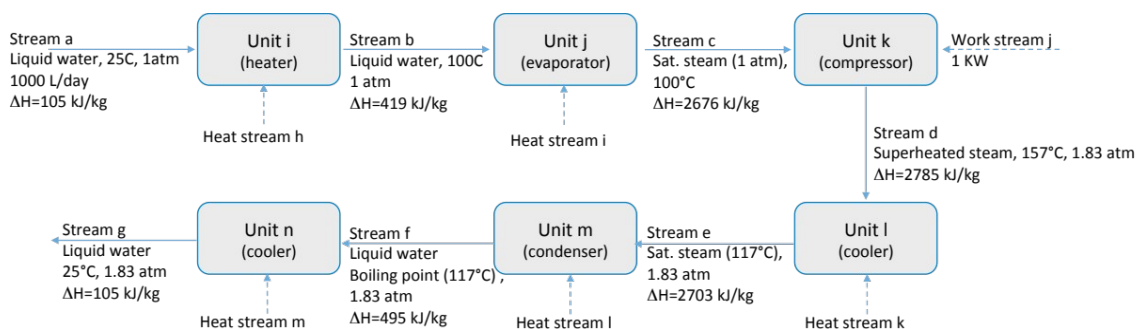


# ChE-304 Problem Set 7

Week 8

## Problem 1

Below, the flowsheet of the slingshot process is shown with values for enthalpy.



Note: These enthalpies are based on an enthalpy of zero for liquid water at 0°C, which makes the numbers manageable. If you want to use numbers corresponding to last week's problem set, you need to add -15'880 kJ/kg to the numbers above.

Can you draw hot and cold composite curves for this process?

What is the minimum approach temperature (assuming no hot utility is provided)? Is this reasonable?

At what temperature would the liquid exit the process (temperature of stream g) if the slingshot was to be completely autonomous (no hot or cold utility)?

A grid is included to help you draw the composite curves.

**Solution:**

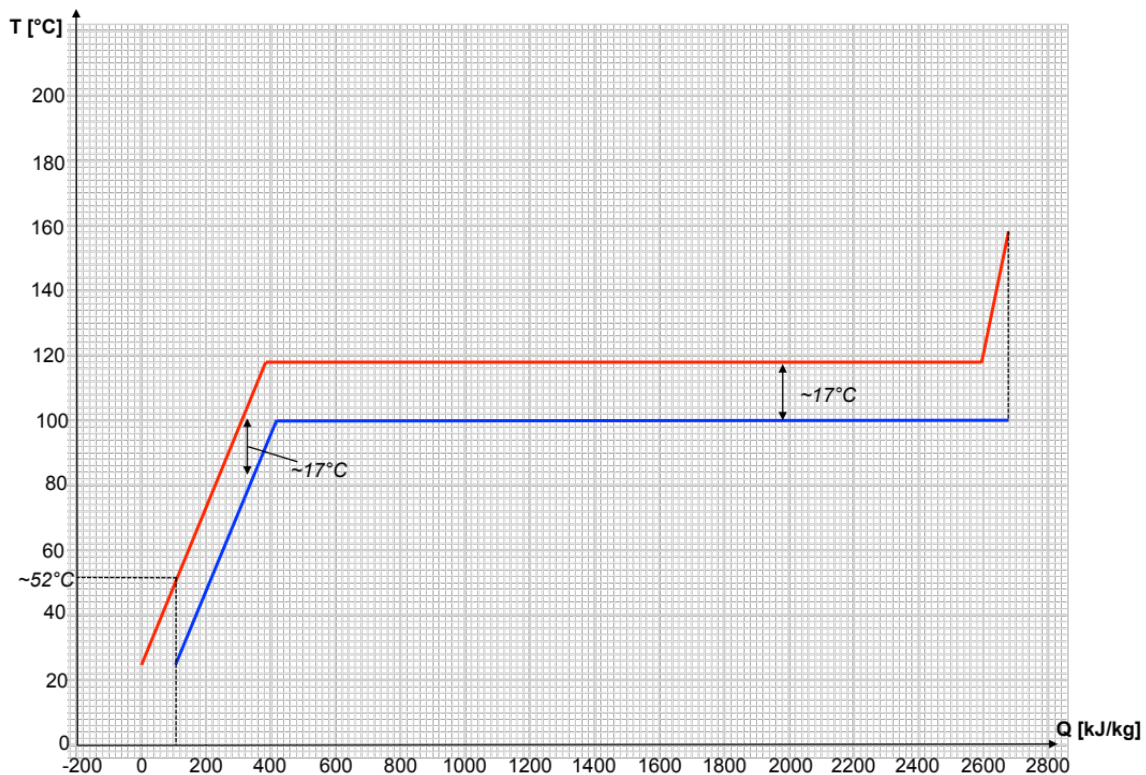
Unit i:  $T_a = 25^\circ\text{C}$   $T_b = 100^\circ\text{C}$   $Q = 419 - 105 = 314 \text{ kJ/kg}$  → cold stream

Unit j:  $T_b = 100^\circ\text{C}$   $T_c = 100^\circ\text{C}$   $Q = 2676 - 419 = 2257 \frac{\text{kJ}}{\text{kg}}$  → cold stream

Unit l:  $T_d = 157^\circ\text{C}$   $T_e = 117^\circ\text{C}$   $Q = 2703 - 2785 = -82 \frac{\text{kJ}}{\text{kg}}$  → hot stream

Unit m:  $T_e = 117^\circ\text{C}$   $T_f = 117^\circ\text{C}$   $Q = 495 - 2703 = -2208 \frac{\text{kJ}}{\text{kg}}$  → hot stream

Unit n:  $T_f = 117^\circ\text{C}$   $T_g = 25^\circ\text{C}$   $Q = 105 - 495 = -390 \frac{\text{kJ}}{\text{kg}}$  → hot stream



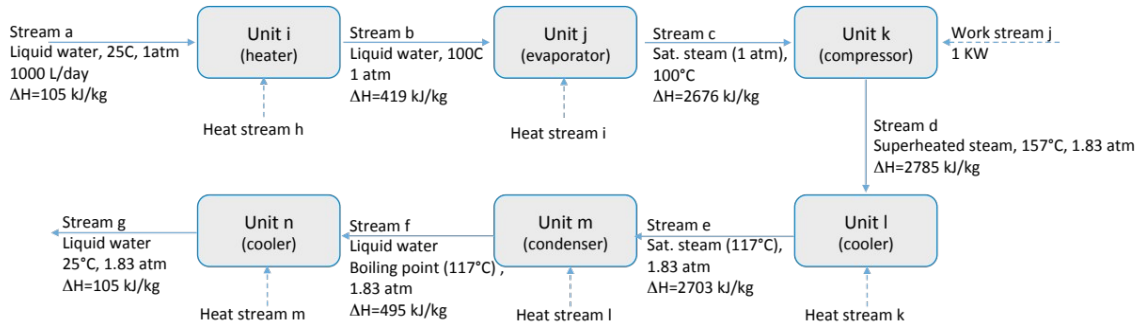
The minimum approach temperature is around  $17^\circ\text{C}$  both during the boiling/condensing phase and the liquid exchange phase. In both cases, we are well above the recommended minimum approach temperature making this process completely feasible. Notice that there is no true pinch point here because we have too much heat at high temperature!

If we cool the hot stream all the way to  $25^\circ\text{C}$ , we would need an external cold utility. To avoid using a cold utility, we need the end of the hot stream to be aligned with the start of the cold stream. By interpolation, we can determine that this alignment would correspond to an exit temperature of about  $52^\circ\text{C}$ .

That being said, in this system, we have assumed that there would be no heat losses to the environment in the process. Since there are likely to be some losses, this final temperature would probably be lower in practice.

## Problem 2

Again, we begin with our process.



Note: These enthalpies are based on an enthalpy of zero for liquid water at 0°C, which makes the numbers manageable. If you want to use numbers corresponding to last week's problem set, you need to add -15'880 kJ/kg to the numbers above.

Perform a heat cascade calculation. Specifically, calculate the  $T_s$ ,  $T_s'$ ,  $T_{f,i}$  and  $GCC$  matrices (see pages 31 and 32 in the notes). From the  $GCC$  matrix can you sketch the grand composite curve?

*Note: Because there is excess heat in this process, the initially constructed grand composite curve will be above zero and will not require the usual translation to set the pinch point at zero.*

### Solution:

Let's start with the  $T_s$  matrix (page 31 in the notes):

$$T_s = \begin{bmatrix} T_{i,i} & T_{i,out} & Q_i \\ T_{j,i} & T_{j,out} & Q_j \\ \dots & \dots & \dots \end{bmatrix} = \begin{bmatrix} 25 & 100 & 314 \\ 100 & 100 & 2257 \\ 157 & 117 & -82 \\ 117 & 117 & -2208 \\ 117 & 25 & -390 \end{bmatrix}$$

Here, to find  $T_s'$ , we will apply a really small temperature change during phase change so as not to have an infinite  $C_p$  (just a really large one). You are free to use something that makes sense or the method proposed in the course. For simplicity, I choose an increase of 0.01°C. In addition, I am correcting the temperatures according to:

$$T_i' = T_i + \frac{\Delta T_{min}}{2} \text{ for } Q_i > 0 \text{ (cold streams)}$$

$$T_i' = T_i - \frac{\Delta T_{min}}{2} \text{ for } Q_i < 0 \text{ (hot streams)}$$

This leads to the following matrix:

$$T s' = \begin{bmatrix} T_{i, \in \dot{c}} T_{i, out}^{\dot{c}} & Cp_i \\ T_{j, \in \dot{c}} T_{j, out}^{\dot{c}} & Cp_j \dots \dot{c} \\ \dots & \dot{c} \end{bmatrix} = \begin{bmatrix} 25+4/2 & 100+4/2 & 314/75 \\ 100+2/2 & 100.01+2/2 & 2257/0.01 \\ 157-8/2 & 117-8/2 & 82/40 \\ 117.01-2/2 & 117-2/2 & 2208/0.01 \\ 117-4/2 & 25-4/2 & 390/92 \end{bmatrix} = \begin{bmatrix} 27 & 102 & 4.2 \\ 101 & 101.01 & 2.3 \cdot 10^5 \\ 153 & 113 & 2.1 \\ 116.01 & 116 & 2.2 \cdot 10^5 \\ 115 & 23 & 4.2 \end{bmatrix}$$

We can now create the temperature interval matrix (see notes page 31):

$$T \int \dot{c} = \begin{bmatrix} T_1^{\dot{c}} & T_2^{\dot{c}} \\ T_2^{\dot{c}} & T_3^{\dot{c}} \\ \dots & \dots \end{bmatrix} = \begin{bmatrix} 153 & 116.01 \\ 116.01 & 116 \\ 116 & 115 \\ 115 & 113 \\ 113 & 102 \\ 102 & 101.01 \\ 101.01 & 101 \\ 101 & 27 \\ 27 & 23 \end{bmatrix} \dot{c}$$

Note: to check that you accounted for every temperature interval, the number of lines in the matrix should be equal to: (the number of unique temperatures) - 1. Here this corresponds to 10-1=9.

Now, let's calculate the vector  $Q$  (the calculation for each element of this matrix is shown in the notes on page 32). The sum of  $Cps$  ( $\sum (Cp_{i,cold} - Cp_{i,hot})$ ) corresponds to all the  $Cp$  values that are relevant for that temperature interval.

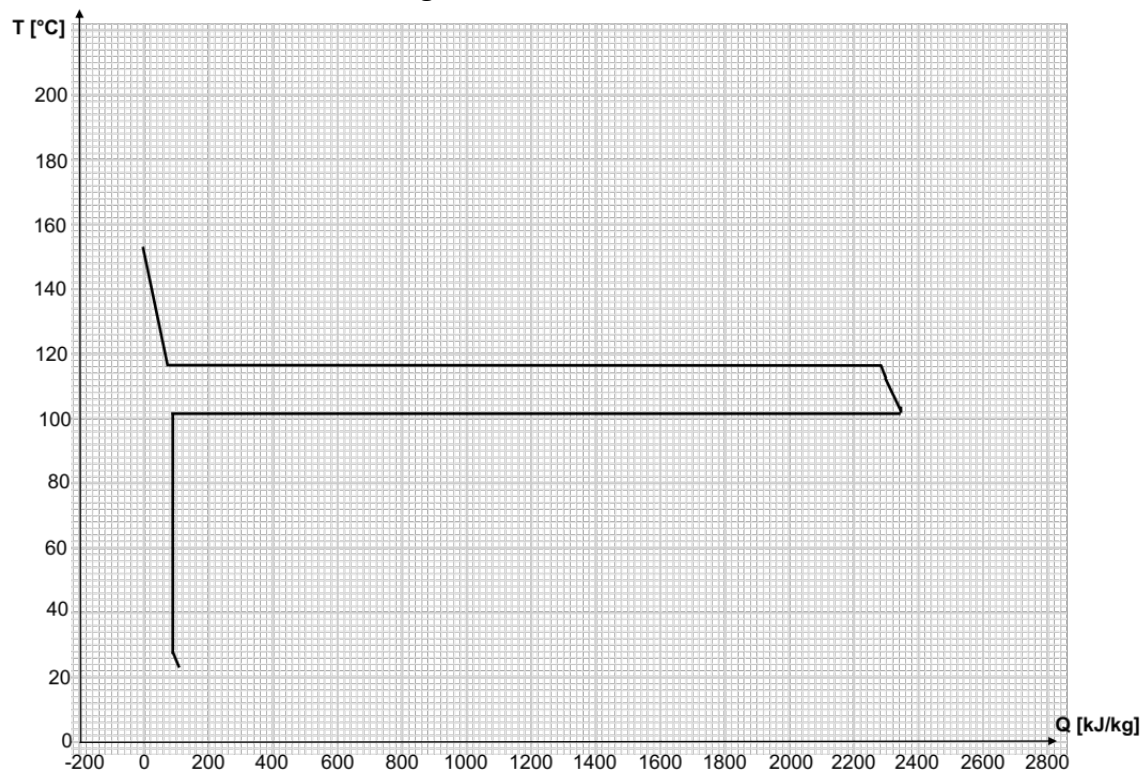
$$Q = \begin{bmatrix} \sum (-Cp_{i,cold} + Cp_{i,hot})(T_1^{\dot{c}} - T_2^{\dot{c}}) \\ \sum (-Cp_{i,cold} + Cp_{i,hot})(T_2^{\dot{c}} - T_3^{\dot{c}}) \\ \dots \end{bmatrix} = \begin{bmatrix} (2.1)(153-116.01) \\ (2.1+2.2 \cdot 10^5)(116.01-116) \\ (2.1)(116-115) \\ (2.1+4.2)(115-113) \\ (4.2)(113-102) \\ (-4.2+4.2)(102-101.01) \\ (-4.2-2.3 \cdot 10^5+4.2)(101.01-101) \\ (-4.2+4.2)(101-27) \\ (+4.2)(27-23) \end{bmatrix} = \begin{bmatrix} 77.7 \\ 2208 \\ 2.1 \\ 12.6 \\ 46.2 \\ 0 \\ -2257 \\ 0 \\ +16.8 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 \\ 77.7 \\ 2285.7 \\ 2287.8 \\ 2300.4 \\ 2346.6 \\ 2346.6 \\ 89.6 \\ 89.6 \\ 106.4 \end{bmatrix}$$

Therefore,  $\min(Q_{c,k})=0$  This is a special case where there is “too much” heat available (we don’t need external heat) and the minimum is the start of the heat cascade. Therefore we add 0. Note that because of this, there is no “pinch point”.

$$GCC = \begin{bmatrix} 153 & 0 \\ 116.01 & 77.7 \\ 116 & 2285.7 \\ 115 & 2287.8 \\ 113 & 2300.4 \\ 102 & 2346.6 \\ 101.1 & 2346.6 \\ 101 & 89.6 \\ 27 & 89.6 \\ 23 & 106.4 \end{bmatrix}$$

The GCC looks like the following:



### Problem 3

Can you use your calculations for the grand composite curve in problem 3 to calculate the exact temperature at which the water should exit in order to have no cooling requirement (in a more exact way than the graphical method employed in Problem 1).

#### Solution:

If no cooling utility is left over it means the GCC needs to end at 0, instead of 106.4:

$$GCC = \begin{bmatrix} 153 & 0 \\ 116.01 & 77.7 \\ 116 & 2285.7 \\ 115 & 2287.8 \\ 113 & 2300.4 \\ 102 & 2346.6 \\ 101.1 & 2346.6 \\ 101 & 89.6 \\ 27 & 89.6 \\ 23 & 106.4 \end{bmatrix}$$

For this to occur, the cascaded heat of 89.6 kJ/kg needs to be eliminated in the last step of the cumulative heat calculation:

$$Q_c = \begin{bmatrix} 0 \\ 77.7 \\ 2285.7 \\ 2287.8 \\ 2300.4 \\ 2346.6 \\ 2346.6 \\ 89.6 \\ 89.6 \\ 0 \rightarrow target \end{bmatrix}$$

For this to occur the final heat load must be equal to -89.6. We can achieve this by increasing the temperature (previously set at 23°C).

Because it is currently a positive heat duty (heat surplus of  $106.4 - 89.6 = 16.8$ ) that we need to turn into a heat deficit (-89.6), this must mean that  $x$  will be higher than 27°C so that the final interval is between  $x$  and 27°C instead of 27°C and 23°C. At the same time, the previous interval will then be between 101°C and  $x$  instead of 101 and 27°C.

$$Q = \begin{bmatrix} \sum (-C p_{i,cold} + C p_{i,hot}) (T_1^{\dot{c}} - T_2^{\dot{c}}) \\ \sum (-C p_{i,cold} + C p_{i,hot}) (T_2^{\dot{c}} - T_3^{\dot{c}}) \\ \dots \end{bmatrix} = \begin{bmatrix} (2.1)(153 - 116.01) \\ (2.1 + 2.2 \cdot 10^5)(116.01 - 116) \\ (2.1)(116 - 115) \\ (2.1 + 4.2)(115 - 113) \\ (4.2)(113 - 102) \\ (-4.2 + 4.2)(102 - 101.01) \\ (-4.2 - 2.3 \cdot 10^5 + 4.2)(101.01 - 101) \\ (-4.2 + 4.2)(101 - x) \\ (-4.2)(x - 27) \end{bmatrix} = \begin{bmatrix} 77.7 \\ 2208 \\ 2.1 \\ 12.6 \\ 46.2 \\ 0 \\ -2257 \\ 0 \\ -89.6 \end{bmatrix}$$

This would occur for a temperature of  $x = \frac{89.6}{4.2} + 27 = 48.3^\circ\text{C}$ .

However, remember that this temperature is modified by  $\frac{-\Delta T_{min}}{2}$ . Therefore, the actual temperature is  $50.3^\circ\text{C}$ .